

# “Instantaneous superluminality” in a bimetallic wire consisting of a superconducting aluminum wire plated with a thick copper covering

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We shall analyze here the transmission of microwave-frequency electrical pulses through a “bimetallic” wire consisting of superconducting aluminum (Al) wire covered with a thick, nonsuperconducting copper (Cu) sheath that is many skin depths thick. See Figure 1. This is another simple example that will demonstrate the possibility of “instantaneous superluminality” [1] within a “bimetallic” superconducting system [2].

The analysis starts from Maxwell’s modification of Ampere’s law by the displacement current

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \right) , \quad (1)$$

where  $\mu_0$  is the magnetic permeability of free space,  $\mathbf{B}$  is the magnetic induction,  $\mathbf{j}$  is the supercurrent density of Cooper pairs at an arbitrary point within the Al wire, and  $\mathbf{D}$  is the displacement field at the same point. We shall examine circumstances under which the supercurrent density within the Al wire portion of the bimetallic-wire configuration can be cancelled out by the displacement current density, such that

$$\mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{0} \text{ everywhere inside the Al wire .} \quad (2)$$

We shall call this the “cancellation-of-currents condition.” This cancellation-of-currents condition (2) is consistent with the continuity equation, since if one were to take the divergence of (2), one obtains

$$\nabla \cdot \mathbf{j} + \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} = \nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0 . \quad (3)$$

It is clear by inspection by the Maxwell equation (1) that when condition (2) holds, the sources of the magnetic induction  $\mathbf{B}$ , which is generated by the point-by-point *superposition* of the supercurrent density  $\mathbf{j}$  and the displacement current density  $\partial \mathbf{D} / \partial t$ , will vanish, since these two sources for generating the  $\mathbf{B}$

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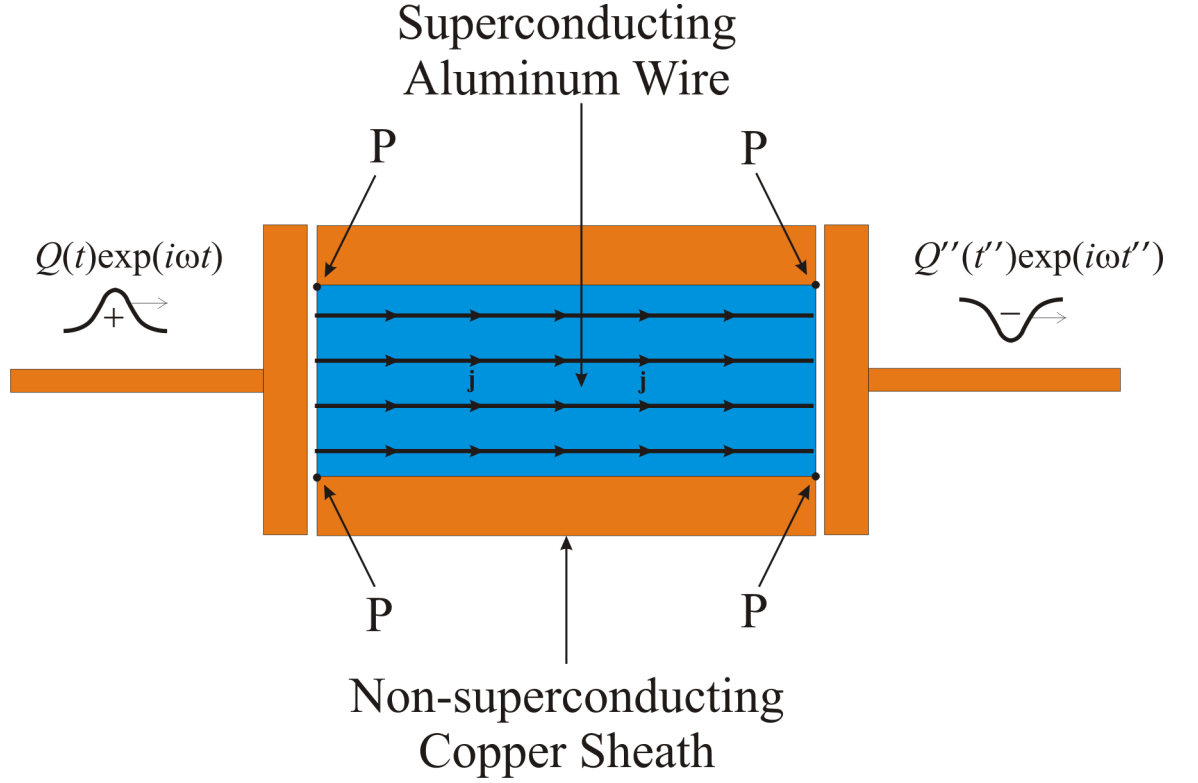


Figure 1: (Not to scale). A simple model of a “bimetallic wire”, consisting of an aluminum wire overcoated by a thick copper sheath, for demonstrating the possibility of “instantaneous superluminality”. A SC aluminum wire (in *blue*) is covered with a thick NSC copper sheath (in *orange*). The equally-spaced horizontal streamlines inside the SC portion of the wire denote the supercurrent density  $\mathbf{j}$ . The wire is excited by a charge pulse  $Q(t)\exp(i\omega t)$  coming in from the left, and the wire transmits a superluminal outgoing pulse  $Q''(t'')\exp(i\omega t'')$  to the right. Points P are places where charges partition into luminal (“type (i)”) surface charges, and superluminal (“type (ii)”) volume charges [3].

field will cancel each other, and therefore that

$$\nabla \times \mathbf{B} = \mathbf{0} ; \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0 . \quad (5)$$

Note that these two equations will be valid even when the supercurrent density  $\mathbf{j}$  and the displacement current density  $\partial \mathbf{D} / \partial t$  may be oscillating at microwave frequencies.

We shall show below that the cancellation-of-currents condition (2) can occur when a superconducting Al wire is coated by a thick nonsuperconducting Cu sheath, as indicated in Figure 1, so that there is an intimate electrical contact between the Al and Cu metals at their interface. This can be implemented by copper-plating aluminum wire. We shall call this a “bimetallic-wire configuration.” The Cu coating on the surface of the Al wire will be chosen to be many skin depths thick, so that at microwave frequencies, currents can flow only on the outer surface of the Cu sheath overlaying the Al wire, and thus these normal electrical currents will be far removed from the interface between the Cu and Al, i.e., they will remain far away from the surface of the Al wire. As a result, any high-frequency  $\mathbf{B}$  field will decay exponentially away from the outer surface of the Cu sheath on the scale of the microwave skin depth of Cu, which is on the scale of microns, and therefore will become vanishingly small at the interface between the Al and the Cu. Hence the Cu sheath effectively forms a tightly fitting Faraday cage over the Al wire.

Therefore the boundary conditions at the surface of the Al wire will be such that a  $\mathbf{B}$  field oscillating at microwave frequencies effectively vanishes at the interface between the Al and the Cu metals. Otherwise, by Faraday’s law of induction, any such high-frequency  $\mathbf{B}$  fields at the interface would generate high-frequency  $\mathbf{E}$  fields within the Cu sheath. However, such electric fields will be shorted out by the high electrical conductivity of the Cu metal immediately surrounding the Al wire. Hence the boundary conditions for the  $\mathbf{B}$  field on the interface between the Al and the Cu, i.e, on the surface of the Al wire, are, to a very good approximation,

$$\mathbf{B} = \mathbf{0} \text{ everywhere on the surface of the Al wire ,} \quad (6)$$

and therefore from Maxwell’s equations (4) and (5), it follows that

$$\mathbf{B} = \mathbf{0} \text{ everywhere inside the volume of the Al wire .} \quad (7)$$

However, from the fact that the supercurrent  $\mathbf{j}$  does not vanish inside the Al wire, it follows that charges must accumulate at the ends of the wire, so that a displacement  $\mathbf{D}$  field must be set up *within* the wire. This displacement field satisfies the Maxwell equation

$$\nabla \cdot \mathbf{D} = \rho \quad (8)$$

where  $\rho$  is the non-vanishing charge density of Cooper pairs accumulating at the two opposite ends of the wire. Note that  $\rho$  and  $\mathbf{D}$  will be oscillating at microwave frequencies.

In order to show that the cancellation-of-currents condition (2) is satisfied inside the bimetallic-wire configuration, let us analyze the equations obeyed by the supercurrent density  $\mathbf{j}$  within the Al wire portion of this configuration. Let us define a time-dependent superflow velocity field  $\mathbf{v}$  of the Cooper pairs within the superconductor through the relationship

$$\mathbf{j} = \rho \mathbf{v} , \quad (9)$$

where the charge density  $\rho$  is related to the complex order parameter  $\psi$  as follows:

$$\rho = q\psi^* \psi \quad (10)$$

where  $q$  is the charge of a Cooper pair.

In order to avoid the enormous Coulomb energies associated with any possible unbalanced charge densities arising from inhomogeneities in the Cooper pair density inside the Al wire, one demands that the charge density of the ionic lattice of the superconductor must be *exactly* compensated by the charge density of the Cooper pairs at every point inside the volume of the superconductor away from the surface. Since we will also assume that the ionic lattice possesses a constant, *homogeneous* density everywhere inside the Al wire, it follows that the Cooper pair charge density  $\rho$  must be a constant of the motion, i.e.,

$$\rho = q\psi^* \psi = \text{constant} . \quad (11)$$

This is consistent with the fact that the ground BCS state of the superconductor corresponds to a uniform charge-density state, and the fact that in first-order perturbation theory, the ground state remains unaltered to lowest order by external perturbations. It follows that

$$\frac{\partial \rho}{\partial t} = 0 \text{ within the volume of the Al wire} . \quad (12)$$

Therefore from the continuity equation (3), it follows that

$$\rho \nabla \cdot \mathbf{v} + \frac{\partial \rho}{\partial t} = \rho \nabla \cdot \mathbf{v} = 0 \quad (13)$$

and therefore that

$$\nabla \cdot \mathbf{v} = 0 . \quad (14)$$

One concludes that the superflow velocity field  $\mathbf{v}$  of the Cooper pairs inside the Al wire of the bimetallic-wire configuration is *incompressible*.

From DeWitt's minimal coupling rule [1], we showed that

$$\mathbf{v} = -\frac{q}{m} \mathbf{A} - \mathbf{h} , \quad (15)$$

where  $\mathbf{v}$  is the superfluid velocity field,  $q$  is the charge and  $m$  is the mass of the Cooper pair, respectively,  $\mathbf{A}$  is the vector potential, and  $\mathbf{h}$  is DeWitt's vector potential. The DeWitt (or "radiation") gauge is being assumed here, with

$$\nabla \cdot \mathbf{h} = \nabla \cdot \mathbf{A} = 0 . \quad (16)$$

Taking the curl of the superfluid velocity field given by (15), one obtains, to a very good approximation,

$$\nabla \times \mathbf{v} = -\frac{q}{m} \nabla \times \mathbf{A} - \nabla \times \mathbf{h} = -\frac{q}{m} \mathbf{B} - \mathbf{B}_G = \mathbf{0}, \quad (17)$$

where  $\mathbf{B} = \nabla \times \mathbf{A} = \mathbf{0}$  is the magnetic field, which vanishes by the solution (7), and  $\mathbf{B}_G = \nabla \times \mathbf{h} = \mathbf{0}$  is the gravito-magnetic field [1], which we shall also assume to vanish everywhere inside the Al wire.

Therefore the superflow velocity field  $\mathbf{v}$  for Cooper pairs inside the Al wire obeys the two equations

$$\nabla \times \mathbf{v} = \mathbf{0}; \quad (18)$$

$$\nabla \cdot \mathbf{v} = 0. \quad (19)$$

One concludes that the superflow of Cooper pairs inside the Al wire of the bimetallic-wire configuration shown in Figure 1 is both *irrotational* and *incompressible*. It follows from (18) that a solution exists of the form

$$\mathbf{v} = \nabla \varphi \quad (20)$$

for some potential function  $\varphi$ , and therefore from (19) that

$$\nabla^2 \varphi = 0. \quad (21)$$

Thus  $\varphi$  obeys Laplace's equation, i.e., the Cooper-pair superflow is *streamline* flow. We know that in the special case of the laminar superflow within a straight pipe of constant cross section, the streamlines are the horizontal straight lines indicated in Figure 1, which satisfy the 1D solution of Laplace's equation, i.e.,

$$\varphi(x, y, z, t) = C(t)x \text{ where } C(t) \text{ is constant independent of } x, y, z, \quad (22)$$

where the  $x$  direction has been chosen to coincide with the direction of the horizontal superflow indicated in Figure 1. The superflow may be *time-dependent* due to the time variations of the incident charge pulse  $Q(t) \exp(i\omega t)$  coming in from the left; nevertheless, there will exist *instantaneous* streamline solutions *everywhere* inside the Al metal having the form given by (22).

Therefore the solution for the supercurrent  $\mathbf{j}$  inside the Al wire of the bimetallic-wire configuration shown in Figure 1 is given by

$$\mathbf{j}(t) = \hat{i} \rho C(t) \text{ everywhere inside the Al wire,} \quad (23)$$

where  $\hat{i}$  is the unit vector in the  $x$  direction of the superflow. As a result, charge will be accumulating at the right end of the Al wire at a rate

$$\frac{dQ}{dt} = jA = \rho C(t)A, \quad (24)$$

where  $j$  is the  $x$  component of the supercurrent, and  $A$  is the cross-sectional area of the Al wire. By charge conservation, there must exist an equal but oppositely

signed charge accumulating at the left end of the Al wire. This will produce a uniform displacement field inside the Al wire.

From the divergence theorem and Gauss's law, i.e., starting from Maxwell's equation

$$\nabla \cdot \mathbf{D} = \rho \quad (25)$$

and applying a “pillbox” argument to the right-end surface of the Al wire [4], it follows that there will be an  $x$  component displacement field  $D$ , with the vector field  $\mathbf{D}$  directed along the  $-x$  axis, produced by the charge  $+Q$  accumulating on the right end of the wire, and, by a similar argument, of the charge  $-Q$  accumulating on the left end of the wire, which is given by

$$D = -\sigma = -\frac{Q}{A} = -\rho \int C(t)dt . \quad (26)$$

at each instant of time  $t$ . Taking the time derivative of this displacement field and comparing it with (23), one obtains the following relationship for the  $x$  components of the supercurrent and the displacement current density:

$$j = -\frac{\partial D}{\partial t} . \quad (27)$$

Therefore, we have verified that, in this special case, the cancellation-of-currents condition (2) is indeed satisfied by the bimetallic-wire configuration shown in Figure 1.

In summary, the only places in the Al wire where charge can accumulate, and therefore where the charge density can change with time, is either at the left end of the wire, where the charge from the incoming Gaussian-envelope microwave pulse is being induced, or at points at the right end of the wire, where this induced charge re-appears in just such way that the *total* charge of the entire Al wire portion of the bimetallic-wire configuration is always *exactly* conserved at every single instant of time  $t$ . This implies that the disappearance of a given electron at the left end of the Al wire is always accompanied by its *simultaneous* reappearance at an arbitrarily far-away point at the right end of the Al wire at *exactly* the same instant of time  $t$ . Otherwise, the principle of charge conservation would be violated.

Following [2], we shall call this counter-intuitive effect “instantaneous superluminality within a bimetallic superconducting wire.” It can be easily verified or falsified in a modification of the simple experiment described in [5], except here one can eliminate the dielectric and the outer conductor of the superconducting-core coaxial cable, and use instead a long coil of bare, copper-plated aluminum wire. Also, one can simply implement the figure ‘8’ gravitational-wave antenna configuration presented in [2] by stripping away the outer layers of the coaxial cable bent into the shape of a figure ‘8’, thus leaving only the bimetallic-wire central core of this cable as the antenna.

Again, it should be emphasized that this superluminal effect does not violate relativistic causality because the incident charge pulse has an *analytic* waveform, for example, a Gaussian pulse, with a finite bandwidth (i.e., with frequencies less

than the BCS gap). There exists no discontinuous “front” within the Gaussian waveform, before which the waveform is *exactly* zero. Such a “front” would contain infinitely high frequency components that would exceed the BCS gap frequency, and thus destroy the superconductivity of the wire. Again, this “instantaneously superluminal” effect has similarities with that of a Gaussian wavepacket tunneling through a tunnel barrier in quantum mechanics, whose early analytic tail contains all the information needed to reconstruct the entire transmitted wave packet, including its peak, earlier in time *before* the incident peak could have arrived at a detector traveling at the speed of light [6].

## References

- [1] S.J. Minter, K. Wegter-McNelly, R.Y. Chiao, Physica E **42** (2010) 234–255.
- [2] We had earlier demonstrated theoretically that “instantaneous superluminality” occurs in the simple example of a pulsed electron beam entering through a grounded copper tube inserted into a hole drilled to the center of an aluminum sphere. See “Figure ‘8’ gravitational-wave antenna using a superconducting-core coaxial cable: Continuity equation and its superluminal consequences”, arXiv:1011.1325.
- [3] For a description of the partitioning of charges into “type (i)” and “type (ii)” charges at the partitioning points P, see “Test of the superluminality of supercurrents induced by a local electric field in a superconducting-core coaxial cable”, arXiv:1011.1326. The easiest way to see this is to imagine an incoming DC charge pulse with  $\omega = 0$  of the Gaussian form  $Q(t) = Q_0 \exp(-t^2/\tau^2)$  entering the capacitor on the left. This charge pulse dumps its charge into the left plate of the capacitor, and thereby induces an opposite charge on the right plate of the capacitor. This induced charge will partition at upper point P into surface charges (normal electrons) that leave the capacitor through its upper surface as a charge pulse travelling *luminally* to the right, and into volume charges (Cooper pairs) that propagates *superluminally* to the right along the horizontal streamlines indicated in Figure 1.
- [4] From Maxwell’s equation (8), it follows that the *normal* component of the displacement field  $\mathbf{D}$  is continuous across a boundary, but not necessarily its *tangential* component.
- [5] “Test of the superluminality of supercurrents induced by a local electric field in a superconducting-core coaxial cable”, arXiv:1011.1326.
- [6] R.Y. Chiao, A.M. Steinberg, in Progress in Optics, Vol. 37, E. Wolf (Ed.), Elsevier, Amsterdam, 1997; R.W. Boyd, D.J. Gauthier, in Progress in Optics, Vol. 43, E. Wolf (Ed.), Elsevier, Amsterdam, 2002.